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**International Baccalaureate**
**Baccalauréat International**
**Bachillerato Internacional**

**MATHEMATICAL STUDIES**
**STANDARD LEVEL**
**PAPER 2**

Thursday 5 May 2011 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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*Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.*

**1.**    *[Maximum mark: 16]*

The heat output in thermal units from burning 1 kg of wood changes according to the wood's percentage moisture content. The moisture content and heat output of 10 blocks of the same type of wood each weighing 1 kg were measured. These are shown in the table.

| Moisture content % (*x*) | 8 | 15 | 22 | 30 | 34 | 45 | 50 | 60 | 74 | 82 |
|---|---|---|---|---|---|---|---|---|---|---|
| Heat output (*y*) | 80 | 77 | 74 | 69 | 68 | 61 | 61 | 55 | 50 | 45 |

(a)    Draw a scatter diagram to show the above data. Use a scale of 2 cm to represent 10 % on the *x*-axis and a scale of 2 cm to represent 10 thermal units on the *y*-axis.    *[4 marks]*

(b)    Write down

    (i)    the mean percentage moisture content, $\bar{x}$ ;

    (ii)    the mean heat output, $\bar{y}$ .    *[2 marks]*

(c)    Plot the point $(\bar{x}, \bar{y})$ on your scatter diagram and label this point M.    *[2 marks]*

(d)    Write down the product-moment correlation coefficient, *r*.    *[2 marks]*

The equation of the regression line *y* on *x* is $y = -0.470x + 83.7$ .

(e)    Draw the regression line *y* on *x* on your scatter diagram.    *[2 marks]*

(f)    Estimate the heat output in thermal units of a 1 kg block of wood that has 25 % moisture content.    *[2 marks]*

(g)    State, with a reason, whether it is appropriate to use the regression line *y* on *x* to estimate the heat output in part (f).    *[2 marks]*

**2.**     *[Maximum mark:  23]*

**Part A**

One day the number of customers at three cafés, "Alan's Diner" ($A$), "Sarah's Snackbar" ($S$) and "Pete's Eats" ($P$) was recorded and are given below.

> 17 were customers of Pete's Eats only
> 27 were customers of Sarah's Snackbar only
> 15 were customers of Alan's Diner only
> 10 were customers of Pete's Eats **and** Sarah's Snackbar **but not** Alan's Diner
> 8 were customers of Pete's Eats **and** Alan's Diner **but not** Sarah's Snackbar

(a)     Draw a Venn Diagram, using sets labelled $A$, $S$ and $P$, that shows this information.     *[3 marks]*

There were 48 customers of Pete's Eats that day.

(b)     Calculate the number of people who were customers of all three cafés.     *[2 marks]*

There were 50 customers of Sarah's Snackbar that day.

(c)     Calculate the total number of people who were customers of Alan's Diner.     *[3 marks]*

(d)     Write down the number of customers of Alan's Diner that were also customers of Pete's Eats.     *[1 mark]*

(e)     Find $n\big[(S \cup P) \cap A'\big]$.     *[2 marks]*


*(This question continues on the following page)*

*(Question 2 continued)*

**Part B**

Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

|  | **Pete's Eats** | **Alan's Diner** | **Sarah's Snackbar** | **Total** |
|---|---|---|---|---|
| **Dissatisfied** | 16 | 8 | 16 | 40 |
| **Satisfied** | 26 | 20 | 34 | 80 |
| **Total** | 42 | 28 | 50 | 120 |

One of the survey forms was chosen at random, find the probability that

(a)    the form showed "Dissatisfied";                                          *[2 marks]*

(b)    the form showed "Satisfied" and was completed at Sarah's Snackbar;      *[2 marks]*

(c)    the form showed "Dissatisfied", given that it was completed at Alan's Diner.    *[2 marks]*

A $\chi^2$ test at the 5 % significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

(d)    Write down the null hypothesis, $H_0$, for the $\chi^2$ test.                *[1 mark]*

(e)    Write down the number of degrees of freedom for the test.                *[1 mark]*

(f)    Using your graphic display calculator, find $\chi^2_{calc}$ .              *[2 marks]*

(g)    State, giving a reason, the conclusion to the test.                      *[2 marks]*

**3.**   *[Maximum mark: 20]*

Consider the function $f(x) = x^3 + \dfrac{48}{x}$, $x \neq 0$.

(a)   Calculate $f(2)$.                                                                    *[2 marks]*

(b)   Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$.   *[4 marks]*

(c)   Find $f'(x)$.                                                                        *[3 marks]*

(d)   Find $f'(2)$.                                                                        *[2 marks]*

(e)   Write down the coordinates of the local maximum point on the graph of $f$.   *[2 marks]*

(f)   Find the range of $f$.                                                               *[3 marks]*

(g)   Find the gradient of the tangent to the graph of $f$ at $x = 1$.                *[2 marks]*

There is a second point on the graph of $f$ at which the tangent is parallel to the tangent at $x = 1$.

(h)   Find the $x$-coordinate of this point.                                          *[2 marks]*

**4.**    *[Maximum mark:  13]*

**Give all answers in this question to the nearest whole currency unit.**

In January 2008 Larry had 90 000 USD to invest for his retirement in January 2011.

He invested  40 000 USD  in  US government bonds which paid  4 % per annum **simple interest**.

(a)    Calculate the value of Larry's investment in government bonds in January 2011.    *[3 marks]*

Larry changed this investment into South African rand (ZAR) at an exchange rate of 1 USD = 18.624 ZAR .

(b)    Calculate the amount that Larry received in ZAR from the exchange.    *[2 marks]*

He changed the remaining 50 000 USD to South African rand (ZAR) in January 2008. The exchange rate between  USD  and  ZAR  was  1 USD = 10.608 ZAR .  There was 2.5 % commission charged on the exchange.

(c)    Calculate the value, **in  USD**, of the commission Larry paid.    *[2 marks]*

(d)    Show that the amount that Larry had to invest is 517 000 ZAR, correct to the nearest thousand ZAR .    *[3 marks]*

In January 2008, Larry deposited this money into a bank account that paid interest at a nominal annual rate of 12 %, **compounded monthly**.

(e)    Find the value of the money in Larry's bank account in January 2011.    *[3 marks]*

**5.**    *[Maximum mark:  18]*

Pauline owns a piece of land ABCD in the shape of a quadrilateral.  The length of BC is 190 m, the length of CD is 120 m, the length of AD is 70 m, the size of angle BCD is 75° and the size of angle BAD is 115°.
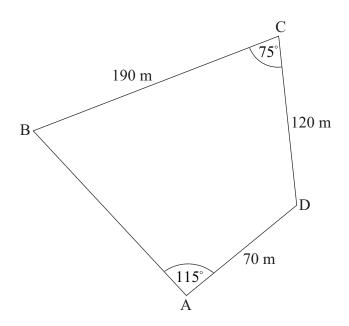
*diagram not to scale*



Pauline decides to sell the triangular portion of land ABD.  She first builds a straight fence from B to D.

(a)    Calculate the length of the fence.                                                    *[3 marks]*

The fence costs 17 USD per metre to build.

(b)    Calculate the cost of building the fence.  Give your answer correct to the nearest USD.                                                                        *[2 marks]*

(c)    Show that the size of angle ABD is 18.8°, correct to three significant figures.    *[3 marks]*

(d)    Calculate the area of triangle ABD.                                              *[4 marks]*

She sells the land for 120 USD per square metre.

(e)    Calculate the value of the land that Pauline sells.  Give your answer correct to the nearest USD.                                                               *[2 marks]*

Pauline invests 300 000 USD from the sale of the land in a bank that pays compound interest compounded annually.

(f)    Find the interest rate that the bank pays so that the investment will double in value in 15 years.                                                                 *[4 marks]*